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Deriving Partial Differential Equation for the Value of *Salam* Contract with Credit Risk

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ABSTRACT

There are many research papers on implementing the *salam* structure in the financial system. This study introduces a mathematical model of *salam* contract with credit risk that can be used as an Islamic financial derivative. It explores the properties of *salam* contract and the credit model that represents it, that is, the structural model with the default event on maturity of the *salam* contract.

Keywords: Salam contract, partial differential equation, credit risk model, Islamic derivative

INTRODUCTION

Salam is a syariah compliant financial contract based on deferred sale. It is an agreement between two parties to carry out a transaction at the future date (maturity) at a price determined today, according to Bacha (2013). The predetermined price or *salam* price $\overline{S(0)}$ has to be paid by the buyer at the initiation of the contract. This is the major difference between *salam* agreement and conventional forwards and futures contract (Hisham & Jaffar, 2014) and believed to help finance small entrepreneur working capital (Bacha, 1999; Yaksick, 1999). According to study done by Yaksick (1999) and Bacha (2013), the payoff or the value of *salam* contract $F(S(t),t)$ at the delivery date $t=T$ is given by:

$$F(S(T),T)=S(T)-\overline{S(0)} \tag{1}$$

where $S(T)$ is the underlying asset spot price at maturity. As we can see, the payoff of *salam* contract is the same with the forwards contract, therefore it closely resembles forwards rather than futures (Bacha, 1999). The *salam* agreement gains if the underlying price at maturity is priced greater than

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the *salam* price $S(t) > \overline{S(0)}$ and losses when the spot price at maturity is less than the predetermined *salam* price $S(t) < \overline{S(0)}$.

As there exist some similarities between *salam* and conventional futures and forwards contract, a comparative research has been done. Ebrahim and Rahman (2005) have investigated the pareto optimality between synthetic futures contract and *salam* contract. Their synthetic future contract was formed by combining future contract on Islamic permissible commodities and Islamic cost-plus sale contract (Bai Murabaha). Their proposed synthetic futures have dominated *salam* contract on efficiency and welfare issues. However, the result is contrary to the intuition that under competitive markets, arbitrage free first order condition leads to pareto neutral (arbitrage free) of both contracts. Dali and Ahmad (2005) have proposed the application of *salam* contract in dinar economy as it can reduce price uncertainty and need for advance prepayment of capital. Susanti (2010) has proposed the implementation of *salam* agreement in murabahah financing contract, while Muneeza, Nurul Atiqah Nik Yusuf and Hassan (2011) highlight the possibility of *salam* application in the Malaysian Islamic banking system to assist farmers with working capital.

This study introduces a mathematical model to ascertain a *salam* contract with credit risk and therefore as an alternative Islamic derivative. The paper is organized as follows: 1) The definition and properties of *salam* contract, 2) the appropriate credit risk model that resembles the properties of *salam* structure, 3) the partial differential equation that describes the value of *salam* contract with credit risk.

CREDIT RISK MODEL

Based on prepayment at the beginning of the contract, the *salam* price is expected to be lower than the prevailing spot price at maturity (Bacha, 1999; Yaksick, 1999). Since the buyer has fully paid *salam* price at the beginning of the contract, he is exposed to the possibility of not receiving the goods at maturity from the seller (credit default risk). Hence, a discount from spot price which would compensate for credit risk (Vogel & Hayes, 1998) to overcome the potential default, syariah allows for the buyer to ask for guarantee such as mortgage and collateral (Bacha, 1999). In modeling *salam* contract this study adopts the method used in credit risk models; the structural model and reduced form model. The default in structural model is determined based on the evolution of the firm's structural variables (assets, liabilities and equities), whereby the reduced form model has assumed that default event is controlled by a Poisson process (Liang & Ren, 2007). In this study, we try to focus on the structural model as default risk in the *salam* contact, where the seller failed to deliver the agreed goods at the maturity leading to a default event.

In the structural model proposed by Merton (1974) default only occurs at the maturity of the contract. Using this approach, Jonson and Stulz (1987) have priced the option with default risk by assuming that the option buyer's claim is the only liability. Klein (1996) by assuming that there exist other liabilities other than the buyer claim, describes the option value by the stochastic process of both the option writer 's asset while the default only occurs when option writer's asset falls below the fixed default boundary. Assuming a stochastic interest rate, Klein and Inglis (1999) have modeled a defaultable option. Klein and Inglis (2001) have also modeled a vulnerable European option that allows for the total liabilities of the option writer to depend on the value of the option holder claim.

The second approach is the first passage time model introduced by Black and Cox (1976) where default can occur at any time prior to maturity. The firm will default when the value of the firm's asset crosses the default barrier. Longstaff and Schwartz (1995) and Briys and Varenne (1997) have extended this approach by assuming that the interest rate is stochastic in pricing bonds with credit risk. Liao and Huang (2005) have modeled a vulnerable option that can cause default prior to maturity by considering the correlation between the option writer's asset, the underlying asset and the value of the default-free zero coupon bond. Liang and Ren (2007) introduced a more practical default barrier where the default event occurs based on price fluctuations of the option value. In constructing a *salam* model with credit risk, this study will consider the structural model of the first approach as the default event only occur at the maturity of *salam* contract.

THE MODEL

In this section, the assumptions underlying Klein and Inglis (2001) for valuing European options subject to credit risk are used with adaptations to incorporate the default mechanism of *salam* contract. It follows the assumptions of Black and Scholes (1973), Merton (1974), Johnson and Stulz (1987), Klein (1996), Lio and Huang (2005) and Liang and Ren (2007).

Assumption 1 (*Salam* contract)

Consider a *salam* contract written on the underlying asset $S(t)$ with predetermined *salam* price $\overline{S(0)}$ and maturity time T. We assume that the underlying asset at time t, $S(t)$ follows the diffusion process by the following stochastic differential equation:

$$dS(t) = \mu(S)S(t)dt + \sigma(S)S(t)dW(t) \tag{2}$$

where $\mu(S)$ is the instantaneous expected return on the underlying asset, $\sigma(S)$ is the volatility of the underlying asset and $W(t)$ is a standard Wiener process for underlying asset.

Assumption 2 (*Salam* writer's asset)

The *salam* contract is issued by the *salam* writer. Let $V(t)$ denote the market value of all *salam* writer's assets at time t. The dynamic of $V(t)$ is described by:

$$dV(t) = \mu(V)V(t)dt + \sigma(V)V(t)dZ(t) \tag{3}$$

where $\mu(V)$ is the instantaneous expected return on the assets of *salam* writer, $\sigma(V)$ is the volatility of the *salam* writer's assets and $Z(t)$ is the standard wiener process for *salam* writer's asset. Since $Z(t)$ has the standard Brownian motion in the same space with $W(t)$, hence they are correlated with:

$$\text{cov}(dW(t), dZ(t)) \equiv E[dW(t) \cdot dZ(t)] = \rho(S,V)dt \tag{4}$$

Where $\rho(S,V)$ is the correlation coefficient between the two Brownian motion with $|\rho(S,V)| < 1$.

Assumption 3

It is assumed that both $S(t)$ and $V(t)$ are traded. Although $V(t)$ is not directly traded however the market value of the assets of counterparty behaves as if it is a traded asset (Klein, 1996).

Assumption 4

Trading of assets happen in continuous time and the market for traded assets are perfect with no transaction costs and taxes (Black & Scholes, 1973; Merton, 1974).

Assumption 5

It is possible to have unrestricted borrowing and lending of funds at the same instantaneous risk-free profit rate, r (Black & Scholes, 1973; Merton, 1974). However, to eliminate the element of *riba* in this *salam* contract transaction, we will replace it with the Islamic interbank rate.

Assumption 6

Claim of the *salam* contract holder is the only liability of *salam* writer.

Assumption 7

Due to the prepayment of predetermined *salam* price $\overline{S(0)}$ at the beginning of the contract $t = 0$, *salam* holder is exposed to the risk that the seller won't deliver the agreed underlying asset at the maturity. Hence, in order to overcome the potential of default from the seller, the *salam* writer's assets will take as the collateral of this *salam* contract.

Assumption 8

Salam contract matures at time T at which it pays $S(T) - \overline{S(0)}$ if the *salam* writer solvent where his total assets value greater than the *salam* holder claim at maturity $(V(T) > S(T) - \overline{S(0)})$. If *salam* writer failed to deliver the claim of *salam* holder at maturity (default) $(V(T) < S(T) - \overline{S(0)})$, *salam* holder will receive the assets of *salam* writer which have the value of $V(T)$ at time T.

Assumption 9

Underlying asset $S(t)$ and *salam* writer asset $V(t)$ cannot take negative value

$$S(t) \geq 0, \qquad V(t) \geq 0$$

This is because the value of any assets can only take nonnegative values (Merton, 1974).

Assumptions 6, 7 and **8** are due to the unique properties of *salam* contract.

RISK NEUTRAL VALUATION

In this section, the stochastic differential equation (2) and (3) for the dynamic of *salam* contract value can be combined with a replication portfolio to eliminate the uncertainty represented by $dW(t)$ and $dZ(t)$. This results with a dynamic partial differential equation for a *salam* contract value which is deterministic. The partial differential equation will give the changes of *salam* contract value with credit risk as a function of underlying asset, *salam* writer's assets and time $F(S(t),V(t),t)$.

Change of Measure

In adopting the risk neutral valuation for valuing the value of *salam* contract, we must transform first the real diffusion process describing the underlying asset and *salam* writer's assets into the risk neutral process. According to Hilliard and Reis (1998) the expected growth of the underlying in the risk neutral world is given by $\mu(S) - \lambda(S)\sigma(S)$ where $\lambda(S)$ is the market price of risk associated with the underlying asset. Since the underlying asset behaves as the traded security that provides a return r then it can be written that:

$$r = \mu(S) - \lambda(S)\sigma(S)$$

Thus,

$$\mu(S) = r + \lambda(S)\sigma(S) \tag{5}$$

$$\lambda(S) = \frac{\mu(S) - r}{\sigma(S)} \tag{6}$$

Substituting (5) and (6) into (2), the following is obtained:

$$dS(t) = rS(t)dt + \sigma(S)S(t)\left(dW(t) + \frac{\mu(S) - r}{\sigma(S)} dt \right) \tag{7}$$

By comparing the terms in (2) and (7), Thus the relationship of Brownian motion for underlying asset in true probability measure $dW(t)$ and martingale probability measure $dW^*(t)$ is given by:

$$dW^*(t) = dW(t) + \frac{\mu(S) - r}{\sigma(S)}dt$$

$$dW(t) \;\; = dW^*(t) - \frac{\mu(S) - r}{\sigma(S)}dt \tag{8}$$

where $\frac{\mu(S) - r}{\sigma(S)}$ deducts the market price per unit of underlying asset risk (Bjerksund, 1991; Cuthbertson & Nitzsche, 2001; Hosseini, 2007). Hence, from (7) and (8), the appropriate risk neutral process for the purpose of valuing the *salam* contract depending on $S(t)$ is given by:

$$dS(t) = rS(t)dt + \sigma(S)S(t)dW^*(t) \tag{9}$$

Based on **Assumption 3**, the *salam* writer's assets have behave like a traded asset, hence the expected growth rate of the *salam* writer's assets in the risk neutral world is given by $\mu(V) - \lambda(V)\sigma(V)$ that can be written as:

$$r = \mu(V) - \lambda(V)\sigma(V)$$

$$\mu(V) = r + \lambda(V)\sigma(V) \tag{10}$$

$$\lambda(V) = \frac{\mu(V) - r}{\sigma(V)} \tag{11}$$

where $\lambda(V)$ is a constant market price of *salam* writer's assets risk. By substituting (10) and (11) into (3) it will get:

$$dV(t) = rV(t)dt + \sigma(V)V(t)\left(dZ(t) + \frac{\mu(V) - r}{\sigma(V)}dt\right) \tag{12}$$

The terms in (3) and (12) are compared, thus the relationship of Brownian motion for *salam* writer's assets in true probability measure $dZ(t)$ and martingale probability measure $dZ^*(t)$ is given by:

$$dZ^*(t) = dZ(t) + \frac{\mu(V) - r}{\sigma(V)}dt$$

$$dZ(t) \;\; = dZ^*(t) - \frac{\mu(V) - r}{\sigma(V)}dt \tag{13}$$

where $\frac{\mu(V) - r}{\sigma(V)}$ deducts the market price per unit of *salam* writer's assets risk. Therefore, the risk neutral process of *salam* writer's assets that will use for valuing *salam* contract is given by:

$$dV(t) = rV(t)dt + \sigma(V)V(t)dZ^*(t) \tag{14}$$

Both standard Brownian motion process $dW^*(t)$ and $dZ^*(t)$ are correlated with:

$$\text{cov}(dW^*(t), dZ^*(t)) \equiv E[dW^*(t) \cdot dZ^*(t)] = \rho(S,V)dt \tag{15}$$

The Process of $\ln S(t)$ and $\ln V(t)$

Based on the conjecture that the underlying asset has a log normal stationary distribution (Wilmott, 2007), defining $X = \ln S(t)$ where $X(S(t))$. Then, we have:

$$\frac{dX}{dS(t)} = \frac{1}{S(t)} \tag{16}$$

$$\frac{d^2X}{dS^2(t)} = -\frac{1}{S^2(t)} \tag{17}$$

By applying one-dimensional Itô lemma (Segupta, 2005) on $X(S(t))$, we get:

$$dX = \frac{\partial X}{\partial S(t)} dS(t) + \frac{1}{2!}\left[\frac{\partial^2 X}{\partial S^2(t)} (dS(t))^2\right] + \ldots \tag{18}$$

where $(dt)^2 = 0$, $dtdW^*(t) = 0$ and $dW^*(t)dW^*(t) = dt$. By substituting the risk neutral process of underlying asset in equation (9) into the Itô lemma process in (16), (17) and (18) we have:

$$d[\ln S(t)] = \left(r - \frac{1}{2}\sigma^2(S)\right)dt + \sigma(S)dW^*(t) \tag{19}$$

where the process of $\ln S(t)$ can be described by equation (19). For *salam* writer's assets, based on the same conjecture as above whereby, it is assumed that *salam* writer's assets have a log normal stationary distribution. Therefore, defining $Y = \ln V$ where $Y(V(t))$

$$\frac{dY}{dV(t)} = \frac{1}{V(t)} \tag{20}$$

$$\frac{d^2Y}{dV^2(t)} = -\frac{1}{V^2(t)} \tag{21}$$

Then, applying one-dimensional Itô lemma on $Y(V(t))$ we have:

$$dY = \frac{\partial Y}{\partial V(t)} dV(t) + \frac{1}{2!}\left[\frac{\partial^2 Y}{\partial V^2(t)} (dV(t))^2\right] + \ldots \tag{22}$$

where $(dt)^2 = 0$, $dtdZ^*(t) = 0$ and $dZ^*(t)dZ^*(t) = dt$. By substituting the risk neutral process of *salam* writer's assets in equation (14) into the Itô lemma process in (20), (21) and (22) we have:

$$d[\ln V(t)] = \left(r - \frac{1}{2}\sigma^2(V)\right)dt + \sigma(V)dZ^*(t) \tag{23}$$

Therefore, the process of $\ln V(t)$ can be described by equation (23). Based on equation (19) and (23), it is shown that $\ln S(t)$ and $\ln V(t)$ follow log normal diffusion process.

Partial Differential equation Governing the Salam Contract Value

When both underlying asset $S(t)$ and *salam* writer's assets $V(t)$ are shown to follow log normal diffusion process, it is possible to construct a perfect hedge portfolio of *salam* contract. There are several methods that can be used to form the partial differential equation for the price of the derivative.

Merton (1974) has constructed a replication portfolio that contains of three securities which are the firm, particular security and the riskless debt on pricing the corporate debt. Hilliard and Reis (1998) have been using the same approach as Hull and White (1987) and Scott (1987) by forming a non-arbitrage portfolio that consists of two futures contracts on different maturities and the spot commodity. Cuthbertson and Nitzsche (2001) and Willmott (2007) in their studies have constructed a synthetic portfolio consisting of underlying asset and bond or some derivative to eliminate the underlying uncertainty and then forming a dynamic partial differential equation for the price of derivative.

However, in forming the partial differential equation describing the *salam* contract value, this study adopts the same method in contingent claim analysis as in Gibson and Schwartz (1990), Bjerksund (1991), Hosseini (2007) and Tassis (2013). By invoking the assumption that the price of contingent claim $P(S(t), V(t), t)$ is twice continuously differentiable function of $S(t)$ and $V(t)$ (Gibson & Schwartz, 1990; Bjerksund, 1991), the instantaneous change value can be formed by applying the multi-dimensional Itô lemma (Segupta, 2005) given by:

$$\begin{aligned} dP = {} & \frac{\partial P}{\partial S(t)}dS(t) + \frac{\partial P}{\partial V(t)}dV(t) + \frac{\partial P}{\partial t}dt + \frac{1}{2!}[\frac{\partial^2 P}{\partial S^2(t)}(dS(t))^2 + \frac{\partial^2 P}{\partial S(t)\partial V(t)} \\ & dS(t)dV(t) + \frac{\partial^2 P}{\partial S(t)\partial t}dS(t)dt + \frac{\partial^2 P}{\partial V^2(t)}(dV(t))^2 + \frac{\partial^2 P}{\partial V(t)\partial S(t)}dV(t)dS(t) \\ & + \frac{\partial^2 P}{\partial V(t)\partial t}dV(t)dt + \frac{\partial^2 P}{\partial^2 t}(dt)^2 + \frac{\partial^2 P}{\partial t\partial S(t)}dtdS(t) + \frac{\partial^2 P}{\partial t\partial V(t)}dtdV(t)] + \ldots \end{aligned} \tag{24}$$

where $(dt)^2 = 0$, $dtdW^*(t) = 0$, $dtdZ^*(t) = 0$, $dW^*(t)dZ^*(t) = \rho(S,V)dt$, $(dW^*(t))^2 = dt$ and $(dZ^*(t))^2 = dt$. By substituting the correlated risk neutral process of underlying asset and *salam* writer asset as in (9) and (14) into the multi-dimensional Itô lemma (24), we have:

$$dP = [(rS(t)\frac{\partial P}{\partial S(t)} + rV(t)\frac{\partial P}{\partial V(t)} + \frac{\partial P}{\partial t} + \frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 P}{\partial S^2(t)}$$
$$+ \rho(S,V)S(t)V(t)\sigma(S)\sigma(V)\frac{\partial^2 P}{\partial S(t)\partial V(t)} + \frac{1}{2}V^2(t)\sigma^2(V)$$
$$\frac{\partial^2 P}{\partial V^2(t)}]dt + \sigma(S)S(t)\frac{\partial P}{\partial S(t)}dW^*(t) + \sigma(V)V(t)\frac{\partial P}{\partial V(t)}dZ^*(t) \tag{25}$$

Equation (25) above describes the instantaneous change of the value of contingent claim $dP(S(t),V(t),t)$. Based on perfect market assumption which imply in the absence of arbitrage and considering the interest rate is not stochastic, the portfolio strategy $W(t) = W^*(t)$ and $Z(t) = Z^*(t)$ are chosen such that the coefficient of $dW(t)$ and $dZ(t)$ are always zero (Merton, 1974; Cuthbertson & Nitzsche, 2001). This strategy can be shown as in our risk neutral process in equation (9) and (14). Therefore, the value of the contingent claim will only be left with the deterministic term as below:

$$dP = [(rS(t)\frac{\partial P}{\partial S(t)} + rV(t)\frac{\partial P}{\partial V(t)} + \frac{\partial P}{\partial t} + \frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 P}{\partial S^2(t)}$$
$$+ \rho(S,V)S(t)V(t)\sigma(S)\sigma(V)\frac{\partial^2 P}{\partial S(t)\partial V(t)} + \frac{1}{2}V^2(t)\sigma^2(V)\frac{\partial^2 P}{\partial V^2(t)}]dt \tag{26}$$

By applying no arbitrage argument, set up a risk-free portfolio as:

$$dP = rPdt \tag{27}$$

Equation (27) above describes that, if we have a completely risk free change dP in the portfolio value P then it must be the same as the growth that we would get if we put the equivalent amount of cash in the risk-free interest bearing account. Hence, by substituting (26) into (27), the market value of that contingent claim must satisfy the partial differential equation as below:

$$rS(t)\frac{\partial P}{\partial S(t)} + rV(t)\frac{\partial P}{\partial V(t)} + \frac{\partial P}{\partial t} + \frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 P}{\partial S^2(t)}$$
$$+ \rho(S,V)S(t)V(t)\sigma(S)\sigma(V)\frac{\partial^2 P}{\partial S(t)\partial V(t)} + \frac{1}{2}V^2(t)\sigma^2(V)\frac{\partial^2 P}{\partial V^2(t)} - rP = 0 \tag{28}$$

According to Cuthberson and Nitzsche (2001) and Hosseini (2007), the change in the value of replication portfolio must match with the change in the value of derivative contract:

$$dP(S(t),V(t),t) = dF(S(t),V(t),t) \tag{29}$$

Hence, under the same market condition and applying the same no arbitrage argument as in (26) and (27), it can be shown that the value of *salam* contract with credit risk $F(S(t), V(t), t)$

must satisfies the following two-dimensional partial differential:

$$rS(t)\frac{\partial F}{\partial S(t)}+rV(t)\frac{\partial F}{\partial V(t)}+\frac{\partial F}{\partial t}+\frac{1}{2}S^2(t)\sigma^2(S)\frac{\partial^2 F}{\partial S^2(t)}$$
$$+\rho(S,V)S(t)V(t)\sigma(S)\sigma(V)\frac{\partial^2 F}{\partial S(t)\partial V(t)}+\frac{1}{2}V^2(t)\sigma^2(V)\frac{\partial^2 F}{\partial V^2(t)}-rF=0 \quad (30)$$

Under equivalent martingale theory (Bjerksund, 1991; Cuthberson & Nitzsche, 2001) the current value of *salam* contract with credit risk of a claim on a future delivery at time T is given by:

$$F(S(t),V(t),t)=e^{-r(T-t)}E^*(S(t),V(t),t)[\text{payoff at } T] \quad (31)$$

where $E^*(S(t),V(t),t)[.]$ is the expectation taken under equivalent martingale probability measure. Therefore, equation (31) can be solved by considering the payoff of *salam* contract with credit risk at maturity as in equation (1), the claim in **Assumption 6**, the collateral as in **Assumption 7**, the default condition in **Assumption 8** and the non-negativity condition as in **Assumption 9**.

CONCLUSION

In constructing the partial differential equation to describe the dynamic behavior of *salam* contract value and credit risk, the diffusion process of the underlying asset and the *salam* writer's assets are reduced to risk neutral process. The properties of underlying asset and *salam* writer's assets process are investigated and a hedging portfolio constructed to eliminate uncertainty. The partial differential equation that describes the dynamic of *salam* contract value with credit risk was constructed based on perfect market conditions. This study aims to solve the two dimensional partial differential equation as in (30), by considering the payoff equation (31) subjects to **Assumption 6**, **7**, **8** and **9** to provide a model of *salam* contract with credit risk.

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REFERENCES

Bacha, O. I. (1999). Derivative instruments and Islamic finance: some thoughts for a reconsideration. *International Journal of* Islam*ic Financial Services*, *1*(1), 9-25.

Bacha, O. I. (2013). Risk management, derivatives and shariah compliance. In *Proceedings of the 20th National Symposium on Mathematical Sciences, AIP Conference Proceedings* (pp. 17-28). AIP

Bjerksund, P. (1991). *Contingent claims evaluation when the convenience yield is stochastic: analytical results*. Bergen-Sandviken: Institutt for foretaksøkonomi, Institute of Finance and Management Science.

Black, F., & Cox, J. C. (1976). Valuing corporate securities: Some effects of bond indenture provisions. *The Journal of Finance*, *31*(2), 351-367.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, *81*(3), 637-654.

Briys, E., & De Varenne, F. (1997). Valuing risky fixed rate debt: An extension. *Journal of Financial and Quantitative Analysis*, *32*(02), 239-248.

Cuthbertson, K., & Nitzsche, D. (2001). *Financial engineering, derivatives and risk management*. United Kingdom, UK: John Wiley & Sons, Ltd.

Dali, N. R. S. M., & Ahmad, S. (2005). A Review of Forward, Futures, and Options From The Shariah Perspective. "From Complexity to Simplicity". In *Conference on Seminar Ekonomi & Kewangan* Islam (pp.29-30).

Ebrahim, M. S., & Rahman, S. (2005). On the pareto-optimality of the futures contracts over Islamic forward contracts: implications for the emerging muslim economies. *Journal of Economic Behavior and Organization*, *56*(2), 273-295.

Gibson, R., & Schwartz, E. S. (1990). Stochastic convenience yield and the pricing of oil contingent claims. *The Journal of Finance*, *45*(3), 959-976.

Hilliard, J. E., & Reis, J. (1998). Valuation of Commodity Futures and Options under Stochastic Convenience Yields, Interest Rates, and Jump Diffusions in the Spot. *Journal of Financial and Quantitative Analysis, 33*(1), 61-86.

Hisham, A. F. B., & Jaffar, M. M. (2014, December). A review on mathematical methods of conventional and Islamic derivatives. In *International Conference On Quantitative Sciences And Its Applications (ICOQSIA 2014): Proceedings of the 3rd International Conference on Quantitative Sciences and Its Applications* (pp. 308-315). AIP.

Hosseini, M. (2007). *Stochastic Modeling of Oil Futures Prices*. (Master Thesis). Department of Mathematics, Uppsala University.

Hull, J., & White, A. (1987). The pricing of options on assets with stochastic volatilities. *The Journal of Finance*, *42*(2), 281-300.

Johnson, H., & Stulz, R. (1987). The pricing of options with default risk. *The Journal of Finance*, *42*(2), 267-280.

Klein, P. (1996). Pricing Black-Scholes options with correlated credit risk. *Journal of Banking and Finance*, *20*(7), 1211-1229.

Klein, P., & Inglis, M. (1999). Valuation of European options subject to financial distress and interest rate risk. *The Journal of Derivatives*, *6*(3), 44-56.

Klein, P., & Inglis, M. (2001). Pricing vulnerable European options when the option's payoff can increase the risk of financial distress. *Journal of Banking and Finance*, *25*(5), 993-1012.

Liang, G., & Ren, X. (2007). The credit risk and pricing of OTC options. *Asia-pacific Financial Markets*, *14*(1), 45-68.

Liao, S. L., & Huang, H. H. (2005). Pricing Black–Scholes options with correlated interest rate risk and credit risk: an extension. *Quantitative Finance*, *5*(5), 443-457.

Longstaff, F. A., & Schwartz, E. S. (1995). A simple approach to valuing risky fixed and floating rate debt. *The Journal of Finance*, *50*(3), 789-819.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance, 29*(2), 449-470.

Muneeza, A., Yusuf, N. N. A. N., & Hassan, R. (2011). The possibility of application of salam in Malaysian Islamic banking system. *Humanomics*, *27*(2), 138-147.

Scott, L. O. (1987). Option pricing when the variance changes randomly: Theory, estimation, and an application. *Journal of Financial and Quantitative analysis*, *22*(4), 419-438.

Sengupta, A. (2005). *Pricing derivatives: The financial concepts underlying the mathematics of pricing derivatives*. United States: McGraw-Hill.

Susanti, D. O. (2010). Pelaksanaan Perjanjian Pembiayaan Murabahah Dengan Sistem Bai'u Salam (Studi Pada PT. BPRS Daya Artha Mentari Bangil-Pasuruan). *Risalah Hukum: Jurnal Hukum, 6*(2), 96-110.

Tassis, D. (2013). *Pricing Of Commodities Futures: An Empirical Study*. (Master thesis). University of Piraeus, Greece.

Vogel, F. E., & Hayes, S. L. (1998). *Islamic law and finance: religion, risk, and return*. The Netherland: Kluwer Law International.

Wilmott, P. (2007). *Introduces qualitative finance* (2nd Ed.). New Jersey: John Wiley & Sons Ltd.

Yaksick, R. (1999). The Islamic Commodity Trust with Application to Crude Oil Forward Sales. In *Proceedings of the third Harvard University Forum of Islamic finance: Local challenges, global opportunities* (pp.213-222). Center for Middle Eastern Studies, Harvard University.